MTH241 Fall 2024: Quiz 07

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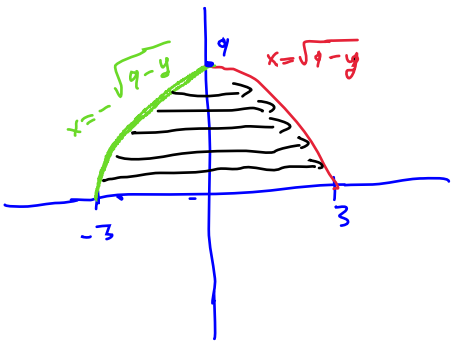
UID:

Closed book, no calculator, show your work clearly.

You don't need to evaluate the integrals.

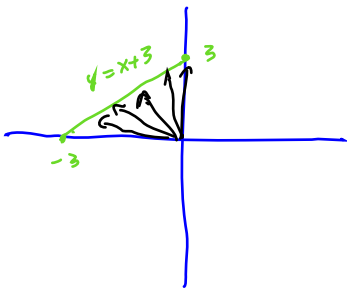
1. (5pt) Let R be the region in the first quadrant bounded by the curve $y = 9 - x^2$. Draw a picture of R and set up an iterated integral in Cartesian coordinates for $\iint_R x dA$ with R as horizontally simple. (Grading: 2pt working; 1pt for drawing; 2pt for integral)



$y = 9 - x^2 \Rightarrow x = \pm\sqrt{9-y}$

$$\iint_R x\, dA = \int_{y=0}^{9} \int_{x=-\sqrt{9-y}}^{x=\sqrt{9-y}} x \; dx\, dy$$

2. (5pt) Let R be the region in the second quadrant bounded by the curve $y = x + 3$, and x and y axes. Draw a picture of R and set up an iterated integral for $\iint_R x^2 + y^2 dA$ with R as polar. (Grading: 2pt working; 1pt for drawing; 2pt for integral)



$y = x+3 \Rightarrow r\cos(\theta) = r\sin(\theta) + 3$

$\Rightarrow r(\cos(\theta) - \sin(\theta)) = 3$

$\Rightarrow \boxed{r = \frac{3}{\cos(\theta) - \sin(\theta)}}$

So $$\iint_R x^2 + y^2 \, dA = \int_{\theta=\frac{\pi}{2}}^{\theta=\pi} \int_{r=0}^{r=\frac{3}{\cos(\theta)-\sin(\theta)}} r^2 \cdot r \; dr\, d\theta$$

Second page: